Period Changes in Pulsating Red Supergiant Stars: A Science and Education Project

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Abstract We describe research done as part of the University of Toronto Mentorship Program, which enables outstanding senior high school students to work on research projects at the university. The students began with extensive background reading on variable stars, and became familiar with various forms of time-series analysis by applying them to a few red supergiant variables in the AAVSO International Database; we report on the results. They also prepared a useful manual for our publicly-available self-correlation analysis software. They undertook an intensive analysis of the period changes in BC Cyg, using the AAVSO and Turner data and the (O–C) method, in the hope that evolutionary period changes could be observed. The (O–C) diagram, however, is dominated by errors in determining the times of maximum, and by the effects of cycle-to-cycle period fluctuations. As a result, the (O–C) method is generally not effective for these stars. We also describe the Mentorship Program and its elements, and reflect on the students' experience.

1. Introduction

Red supergiants are the coolest, largest, most luminous stars, up to a thousand times larger in radius than the Sun. They are massive young stars in the final rapid stages of thermonuclear evolution. They undergo a complex variety of physical processes, including convection, pulsation, and extensive mass loss, which causes most of them to be shrouded in gas and dust. They are also all variable, though not strictly periodic, being classified as SRc if they are semiregular, and Lc if they are not. They vary typically on time scales of hundreds to thousands of days, and amplitudes up to a few magnitudes.

This project was inspired by two recent papers. Kiss *et al.* (2006) (hereinafter KSB) studied forty-eight SRc and Lc stars, using visual observations from the AAVSO International Database. The mean time-span of the data was sixty-one years. Most of the stars showed a period of several hundred days that could be ascribed to radial pulsation. Two or more periods were found in eighteen stars. In some cases, the second period could be an additional radial mode. In other cases, the second period was an order of magnitude longer than the radial period, and could be classified as a "long secondary period," similar to those that have been found in many pulsating red giants, and whose cause is unknown (Wood *et al.* 2004). From the Lorentzian shapes of the individual power spectra, KSB deduced the presence of period "noise," probably due to the interplay between pulsation and convection.

The second paper was by Turner *et al.* (2006) (hereinafter TRBP): they studied BC Cyg using both AAVSO visual data and data obtained from photographic plates in the collections of Harvard College Observatory and Sternberg Astronomical Institute. They concluded, among other things, that the pulsation period of BC Cyg had decreased from 699 to 687 days between 1900 and 2000. This period change, if real, might reflect the rapid evolution of this star.

The primary purpose of this paper was to study the period change in BC Cyg using the (O–C) method. A secondary purpose was to apply other forms of time-series analysis to this and other SRc and Lc stars. An equally important purpose was to provide an authentic research experience for three outstanding senior high school students.

According to the SIMBAD database, BC Cyg (M3.5Ia, HIP 100404, BD +37 3903) is an SRc variable with a photographic range of 11.3–13.8, $V \sim 10.0$, and a period of approximately 700 days; KSB report a period of 720 ± 40 days, and TRBP report a period decreasing from 699 to 687 days. Josselin and Plez (2007) derive the following physical properties for this star: M/M = 20, T_{eff} = 3570K, log R/R = 3.09, $M_{bol} = -8.62$.

2. The University of Toronto Mentorship Program

Authors EF, JG, and BH were participants in the University of Toronto Mentorship Program (UTMP). This program enables outstanding senior high school students to work on research projects at the University. JRP's goal is to provide the students with a reasonably structured research experience that, among other things, enables them to complete a small, self-contained research project that will result in a conference presentation and/or publication. Two other examples of recent UTMP projects are Percy *et al.* (2006), and Percy and Palaniappan (2006). A UTMP co-author of the former paper, Wojciech

Gryc, was a 2008 winner of a Rhodes Scholarship.The UTMP is structured as follows. In May, faculty members submit project descriptions. In August, mentorship program packages, with project descriptions, are sent to all high schools in the Greater Toronto Area. In September, students submit applications: resumé, transcript, references, and statement of interest in one or two projects. In October, faculty select and interview a short list of students; JRP chooses one to three students each year. In November, students begin their project, starting with reading, introduction to data and software—light curves, Fourier, least-squares, self-correlation, (O–C) analysis, and random cycle-to-cycle period fluctuations. They meet with their supervisor every week or two, to discuss both their project and astronomy in general. Often, they attend other astronomical events, such as lectures and star parties. In May, there is a UTMP reception and "research fair," featuring poster presentations on projects from across the University—mostly from the Faculty of Arts and Science. Figure 1 shows co-authors EF, JG, and BH at the research fair. By June, the projects are completed, and prepared for presentation and publication. Often, the students are employed in the summer for a few tens of hours to complete or extend their projects. The UTMP gives students a head start in their research career, which can be very helpful when they undertake their undergraduate studies.

3. Sources of data

Measurements of the SRc and Lc stars came from two sources: (i) Visual measurements from the AAVSO International Database, spanning up to a century; (ii) For BC Cyg, photographic measurements made by DGT from the Harvard Observatory plate collection and by TRBP from the plate collection of the Sternberg Astronomical Institute, spanning just over a century. See TRBP for a discussion of the nature and comparability of these two datasets. As an initial activity, EF, JG, and BH plotted sample light curves, and estimated times of maximum and minimum for several of the larger-amplitude variables.

4. Redetermination of periods by self-correlation

Self-correlation is a simple method of time-series analysis that determines the characteristic time scale and amplitude of the variability, averaged over the dataset. For a discussion of its nature, strengths, and weaknesses, see Percy and Mohammed 2004 and references therein. Our self-correlation software is freely available at:

http://www.astro.utoronto.ca/~percy/index.html

and a new manual for its use, written by co-authors EF, JG, and BH, is available at:

http://www.astro.utoronto.ca/~percy/manual.pdf

As a learning exercise, we began by generating self-correlation diagrams for several stars in KSB's list. The results are as follows:

T Cet showed a time scale of 163 days; the estimated uncertainty is about 3 days. KSB obtained periods of 161 ± 3 and 298 ± 3 days. Co-author DGT separately obtained a period of 288 days by Fourier analysis. The literature periods, as quoted by KSB, are 110, 159, and/or 280 days.

RW Cyg showed a time scale of about 500 days, in agreement with the result of KSB—580 ± 80 days—and the literature periods of 550 and 586 days (KSB).

BC Cyg's self-correlation diagram for the AAVSO data is quite regular, and gives a period of about 700 days, as it does for the combined AAVSO-Turner data. The self-correlation diagram for the Turner data alone is somewhat more scattered.

BU Gem showed a time scale of 2500 days, in good agreement with KSB's result of 2450 ± 750 days. The literature periods are 272 and 1200 days (KSB). There is *weak* evidence for a time scale of 150 days in our self-correlation diagram, but the corresponding amplitude is only 0.01 magnitude.

For XX Per, KSB did not determine the short period. The literature periods are 415 and 4100 days (KSB). Self-correlation analysis gives a slightly irregular period of about 300–350 days (Figure 2).

AH Sco showed a time scale of 380–400 days, approximately half of the period (738 ± 78 days) found by KSB. The self-correlation diagram is complex. The light curve shows evidence of both time scales, at different epochs. Co-author DGT separately obtained a period of 769 days by Fourier analysis. The literature period is 714 days (KSB).

VX Sgr showed a time scale of 750 days, in good agreement with KSB's period of 754 days, though we found possible evidence of weak interference from a time scale of about 250 days. Co-author DGT separately obtained a period of 757 days by Fourier analysis. The literature period is 732 days (KSB).

For CE Tau, KSB did not determine a short period. The literature period is 140–165 days (KSB). Self-correlation analysis gives a well-determined period of 350–375 days (Figure 3). This is suspiciously close to one year, and the amplitude is only 0.02 magnitude, which suggests that the period may be spurious, and due to the well-known "angle effect" in visual photometry. This is caused by the changing relative position of the variable and the comparison stars during the year. CE Tau and a few other stars show small peaks in KSB's Fourier spectra at a period of 365 days.

W Tri showed a time scale of about 107 days, in agreement with KSB's result of 107 ± 6 days. We also found a more complex time scale of about 600 days, in agreement with the period of 590 ± 170 days, found by KSB. Co-author DGT separately obtained a period of 592 days by Fourier analysis.

5. Light curves and times of maximum

The light curves of SRc variables are not regular, as can be seen from those presented by KSB, or from generating light curves using the Light Curve Generator function on the AAVSO website. Figure 4 shows a partial light curve of VX Sgr, for example. It includes one of several epochs at which the amplitude became very small. At these epochs, it is almost impossible to estimate times of maximum or minimum. Omitting these intervals, however, may bias the application of the (O–C) method or of the Eddington-Plakidis method, discussed below. Variable amplitudes *could* be produced by interference between two close periods, in which case there is a characteristic variation in (O–C) across the epoch of minimum amplitude. On the other hand, if the variation in amplitude is caused by an actual variation in pulsation energy, there will be no resulting variation in (O–C).

For VX Sgr, the observations are dense, and the amplitude is up to five magnitudes; it was the largest-amplitude variable in our study. For BC Cyg, the observations are much less dense, and the amplitude is typically one to two magnitudes. So it is even more difficult to determine times of maximum or minimum, especially by eye.

Times of maximum were determined using three methods: eye estimates, the epoch calculator within PERIOD04 (Lenz and Breger 2005), and least-squares fitting of cycles within PERIOD04; the last two are closely related, so we lump them together.

6. Period changes in BC Cyg using the (O–C) method

Figure 5 shows the (O–C) diagram for BC Cyg, using the TRBP data, times of maxima determined by eye, and a period of 693 days. This is probably the most reliable (O–C) diagram, in the sense that it contains fewer gaps, in which the cycle count is uncertain. It is dominated by a cyclic pattern, though the <u(x)> diagram suggests that this pattern is *not* due to random cycle-to-cycle period fluctuations.

Table 1 lists the results of the (O–C) analysis for BC Cyg, using the two datasets, two methods of determining times of maximum, and two possible values of the period. The last column lists the curvature—the coefficient of N^2 in the best-fit parabola, along with its standard error. In no case is the curvature statistically significant (at the 3σ level).

If the period decrease found by TRBP is correct, it would imply a coefficient of –0.114, which is within the error of the determinations in Table 1, and specifically of the (O–C) diagram in Figure 5, namely –0.163 ± 0.259.

We also plotted (O–C) diagrams for RW Cyg, XX Per, VX Sgr, and CE Tau, for which we had AAVSO data only, In each case, the curvature of the best-fit parabola was considerably smaller than its standard error, so we have no positive results to report.

7. Random cycle-to-cycle period fluctuations

These were determined from the times of maximum or minimum using the formalism of Eddington and Plakidis (1929), using an algorithm written by Deepak Chandan (2007) in EXCEL. This determines the average cycle-to-cycle period fluctuation ε, and the average observational error α in determining the times of maximum or minimum. The diagnostic equation is:

$$<u(x)>^2 = 2\alpha^2 + x\varepsilon^2 \tag{1}$$

where *u(x)* is the average difference in (O–C)s which are *x* cycles apart.

As a test of his program, Chandan generated a <u(x)> diagram for VX Sgr, using only well-determined times of maximum; he found an average fluctuation per 743-day cycle of 55.8 days, or about 7 percent. The average observational error in determining the time of maximum or minimum is 120 days, as determined from the intercept, or 78 days as determined from the value of *u(1)*. (We have found that, especially in cases in which the *<u(x)>* diagram is not exactly linear, *u(1)* is a better estimation of α.)

For T Cet, which is a luminosity-class II star, not a supergiant, he found the slope of the line in the Eddington-Plakidis algorithm to be negative, but not significantly different from zero. When one low-weight *u(x)* value was omitted, the slope changed noticeably, but was still not significantly different from zero. There are thus no significant random cycle-to-cycle period fluctuations.

The *<u(x)>* diagram for BC Cyg, using the same data as shown in Figure 5, is shown in Figure 6. The points clearly do not follow a straight line. This is not surprising, given the quasi-cyclic nature of Figure 5. The slope of the best-fit straight line is 142 ± 327, which is not significantly different from zero. The intercept is 14800 ± 3542 but, since a straight line is not a good fit to the data, the intercept is better estimated from *u(1)*. The value of α is 70 ± 20 days, or about 0.1 period.

8. Discussion and conclusions

The interpretation of the (O–C) diagram of pulsating red supergiants appears to depend, to a large extent, on how the times of maximum are measured. This is because most of the visual light curves are not very dense, and the amplitudes are not large—a magnitude or two. We have tried measuring the times by eye, and by fitting techniques such as the least-squares function in PERIOD04. They do not produce identical results. Normally, the "statistical" method will be superior but, in applications such as this one, the human eye/brain system can be a very sophisticated and effective computer. The large values of α found in the <u(x)> analysis are a reflection of this problem. An inherent problem in working with sparse visual data is that different measurements may come from different observers whose eyes have different sensitivities, so there will be both random and systematic errors, whether the times of maximum are measured by eye or by computer. The *u(x)* analyses for T Cet, BC Cyg, and VX Sgr suggest that the average observational error α in measuring the time of maximum or minimum is about 0.1 period, or more.

Therefore it is not possible to measure evolutionary period changes in these stars using the (O–C) method, because the curvature of the (O–C) diagram is not statistically significant. Only with one data set—that of the AAVSO—and with one method of measurement of the times of maxima—by eye—do we find a significant curvature for BC Cyg, but not quite at the 3σ level. The other results in Table 1 do not support this result, including those using the more extensive TRBP data. So it is still possible that BC Cygni has an evolutionary decrease in period; as noted above, the period decrease proposed by TRBP corresponds to a curvature that is within the errors of our determination.

We conclude that the approach of KSB—that is to use the (Lorentzian) profile of the peaks in the power spectrum as an indication of the "scatter" in the period—is a better approach than trying to estimate numerous times of maximum in a sparse, semiregular, low-amplitude light curve, and using the (O-C) method. Note that the *width* of the peaks in the power spectrum provides information about the *uncertainty* in the mean period, as noted by Kwee, van Woerden, Fernie and others many years ago.

The (O–C) diagrams are dominated by cyclic variations, but it is not clear whether these are the same kind of *random* cycle-to-cycle period fluctuations that dominate the (O–C) diagrams of pulsating red giants (Percy and Colivas (1999) and references therein). For BC Cyg, the diagnostic (Figure 6) does not support the period-fluctuation hypothesis, although, for VX Sgr (not shown), it does.

What is the nature of the variability and its complexity? KSB noted that convection could play an important role in producing the variability, and modifying—and perhaps even exciting—the pulsation. Gray (2008) has carried out a detailed long-term spectroscopic study of Betelgeuse (M2 Iab), and compared the spectroscopic variations with AAVSO visual photometry. He concludes that the photometric variability is largely caused by enormous convection cells, with turnover times of about 400 days, comparable to the radial pulsation time scales. KSB estimated the mode lifetimes for Betelgeuse to be about three cycles, after which another convection cell emerges, and chaotic behavior is created in the parameters of the variability. The stochastic nature of the convection, and its driving effect on the pulsation, produces the wandering period, variable amplitude, and variable phase that we observe (Gray 2008).

If the surface of the star is dominated by one or two giant convection cells, and if their lifetime is sometimes comparable with the star's rotation period, then we might expect to see some variability on a rotational time scale. This may cause some of the long-term variability in both the brightness and the phase (as measured by (O–C)) in some of these stars.

Self-correlation analysis is a useful adjunct to Fourier analysis for determining the time scales of these stars, as it has been for other types of semiregular variables. For at least two stars in our dataset, it provides new information about the period of the star.

AAVSO visual observations are essential for understanding these stars. The behavior of these stars is so slow and complex, and there are so many types of long-term variability, that the visual observations provide the only hope for further understanding. The longer the dataset, the better our chance of understanding will be.

AAVSO and other variable star data, and the many available user-friendly data analysis programs, provide a wide range of useful educational resources that enable students to "learn science by doing science" with real data. This is certainly true for both undergraduate students, and for the students in the UTMP. They enrich their education, contribute to our understanding of stars and their evolution, and provide feedback and satisfaction for the hundreds of observers who have contributed to the AAVSO International Database.

9. Acknowledgements

We thank Deepak Chandan, Rohan Palaniappan, and Rajiv Seneviratne for their assistance with various aspects of this project; the organizers of the University of Toronto Mentorship Program (especially Farheen Hasan); the Natural Sciences and Engineering Research Council of Canada, and the Ontario Work-Study Program for research support; and the AAVSO observers and headquarters staff, without whose efforts this project would not be possible. This research has made use of the SIMBAD database, operated at CDS, Strasbourg, France.

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Table 1. (O–C) Analyses of BC Cyg.

Data	*Max/Min Determined By*	*Period (d)*	*Quadratic Coefficient*
AAVSO	*Period04*	693	–2.46 ± 1.38
AAVSO	eye	693	–2.52 ± 0.842
AAVSO	eye	720	0.738 ± 1.35
TRBP	*Period04*	693	0.125 ± 0.388
TRBP	eye	693	0.016 ± 0.184
TRBP	eye	720	0.230 ± 0.399

Figure 1. Co-authors Bernadette Ho, Elena Favaro, and Jou Glasheen, with their poster at the University of Toronto Mentorship Program Research Fair. The white Christmas lights add a festive and somewhat astronomical touch.

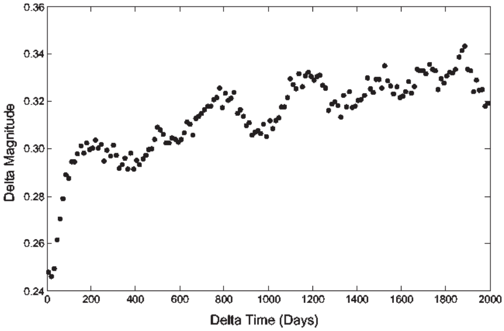

Figure 2. The self-correlation diagram (Δmag versus Δtime) for XX Per. The minima are very shallow, corresponding to amplitudes less than 0.02, and do not repeat in any coherent pattern. The literature period is 415 days, and the self-correlation diagram is not inconsistent with this.

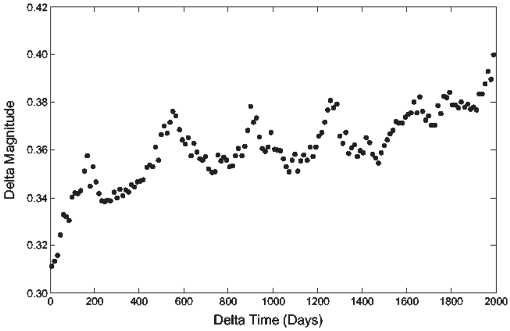

Figure 3. The self-correlation diagram (Δmag versus Δtime) for CE Tau. There are repeating minima at multiples of 375 days, indicating that this is the dominant time scale in the data. The literature period is 140–165 days.

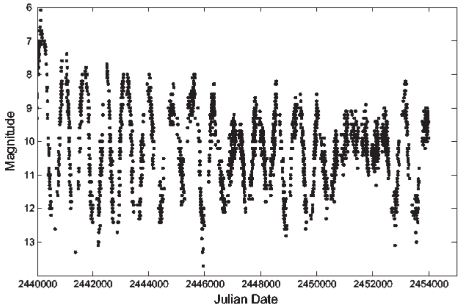

Figure 4. A 15000-day light curve of VX Sgr, based on visual observations from the AAVSO International Database. Note the epoch at which the amplitude becomes small, and the times of maximum and minimum become nearly impossible to determine.

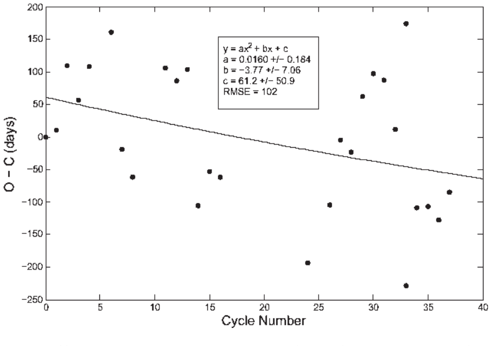

Figure 5. The (O–C) diagram for BC Cyg, using times of maximum determined by eye from the data of Turner *et al.* (2006), and using a period of 693 days. The line shows the best-fit parabola; the curvature, 0.0160 ± 0.184, is not significantly different from zero.

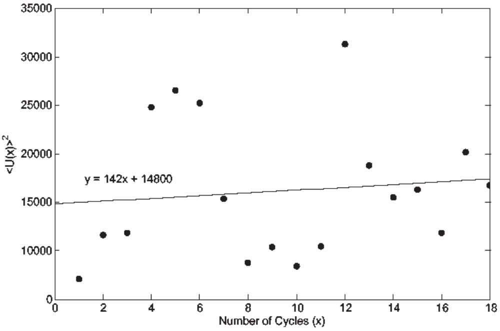

Figure 6. The $<u(x)>^2$ (Eddington and Plakidis 1929) diagram for BC Cyg, based on the (O–C) data shown in Figure 5. The line is the best-fit straight line, but it does not fit the data very well. The nominal slope corresponds to an average cycle-to-cycle fluctuation of 12 days but, as noted in the text, the slope of the line is not significantly different from zero.