Chapter 12: Variable Stars and Phase Diagrams

Introduction

When the same cycle repeats over and over as regularly as clockwork, we refer to this as *periodic* behavior. If we want to know what is happening at any moment, it does not matter which cycle we are observing, because every cycle is exactly the same. What does matter is which *part* of the cycle we are observing. So if a star (or any other phenomenon) is perfectly periodic, then its variation depends only on where it is in its cycle, a quantity called the *phase*.

A good example is an accurate clock. If it is a 24-hour clock (with an AM/PM indicator), it repeats exactly the same behavior, over and over, with a period of 1 day. Each day the clock goes through one cycle, and each cycle is just like every other cycle. If we want to know what the clock reads, we do not need to know which day it is (which cycle it is in), we just need to know the time of day (how far we are into the cycle).

Phase in Cycles

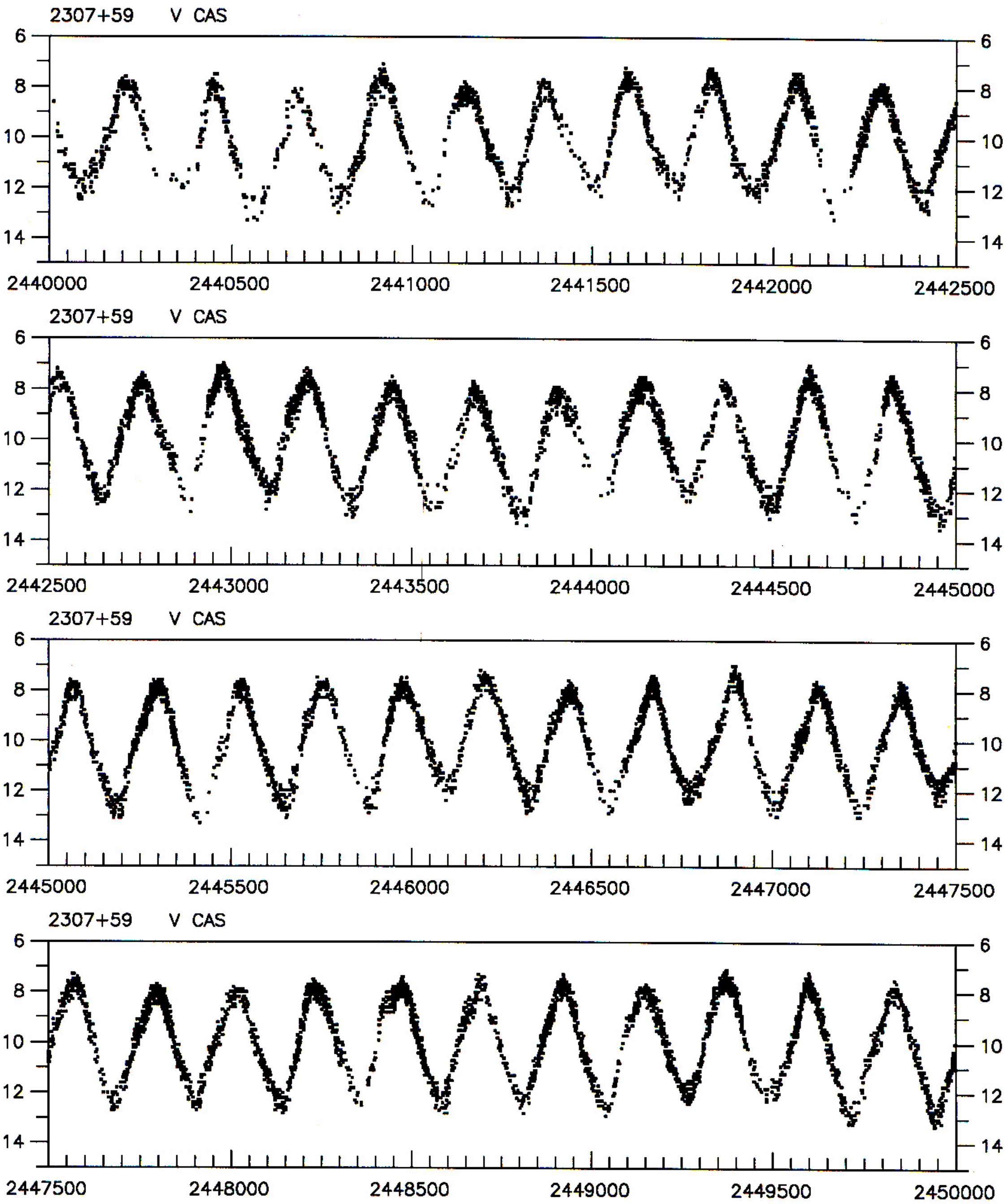
In the case of the clock, we might measure "how far into the cycle" it is in terms of hours and minutes, with the cycle starting at 00:00 and ending at 24:00. Of course, 24:00 (the end of the cycle) is also 00:00 on the *next day*, because the end of one cycle coincides with the beginning of a new one.

However, phase is measured in *cycles*, rather than in hours or minutes. Since phase is measured in cycles, of course a single cycle starts at 0 and ends at 1. In this case, the phase is simply the fraction of the cycle which has been completed so far. Thus a phase of 0.5 corresponds to 0.5 of the way (50%, or halfway) through the cycle, a phase of 0.2 is 20% (one-fifth) of the way through the cycle, etc. A phase of 1 is 100% of the way, the end of the cycle; it is also the beginning of the *next* cycle, so it is phase 0 of the next cycle.

To compute the phase in terms of cycles, we need to know how long each cycle is—in other words, we need to know what the *period* is. For the clock, we can express the phase in hours and minutes. But to express the phase in terms of cycles, we need to know that each cycle (each day) is 24 hours. That way, at noon, when we are 12 hours into the cycle, we know we are 12/24 = 0.5 of the way through, meaning that the phase is 0.5.

Investigation 12.1: Periodic Cycles

Cycles have no real beginning and no real end—they are continuous. Study the continuous light curve for V Cas on the opposite page. Cut out the sections and tape them together to see the behavior of V Cas from JD 2440000 to JD 2450000. If you use just two sections, do you still have a representation of the star's behavior? With just one section? Your instructor will give you a transparency of the same light curve. Can you cut the sections in any place and still have the same behavior pattern? Stack several sections over each other. Determine how small a segment is needed to give the same information and all four segments. What if you only had one cycle of the light curve? What if you start your cycle at maximum? Minimum? Describe your results.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A New Beginning

The clock cycle starts at midnight, because timekeepers have chosen to start each new day at midnight. But this is an arbitrary choice. Other cultures start a new day at sunrise or sunset, rather than at midnight. If the day (the cycle) starts at sunrise (say, 6:00 AM), then we are *not* halfway through the cycle at noon (12:00), so the phase is *not* 0.5. We may have 12 hours on the clock, but because we agree that our cycle *starts* at 6, we are only 12 - 6 = 6 hours into the cycle. That is 6/24 = 0.25 of the cycle (25%, or one fourth), so the phase is 0.25. Therefore, to compute phase we *also* need to know the starting time of the cycle. This is known as the *epoch*. For the clock, the epoch is usually midnight, but some people prefer to start their cycles at some other time.

These two quantities, the *period* and *epoch*, enable us to compute the *phase* at any given time. Suppose the epoch (start of the cycle) is at time t_0, and the period is P. What is the phase at some other time t? First we find how far we are into the cycle, by simply subtracting the starting time:

$$t - t_0$$

This is the phase, in *time* units. To get the phase in units of cycles, we simply divide this by the period:

$$\phi = \frac{t - t_0}{P}$$

The symbol ϕ is the Greek letter “phi,” which is used to represent the phase (in cycles). In the case of the clock, with the cycle starting at 6 AM, the period is $P = 24$ (hours) and the epoch is $t_0 = 6$. At noon ($t = 12$), the phase is:

$$\phi = \frac{t - t_0}{P} = \frac{12 - 6}{24} = \frac{6}{24} = 0.25$$

0 = 1 (yes, zero equals one)

Let's go back to starting each new day at midnight, so for our clock the period is P = 24 12) the phase is 0.5 (halfway), and at the following midnight (t = 24) the phase is 1 (end of the cycle). What about the *following* noon?

In this case the time is t = 36; it has been 36 hours since our "epoch." The phase is:

$$\phi = \frac{36 - 0}{24} = 1.5$$

But something is not right here. We said that phase was "how far along we are in the cycle," and that it did not matter which cycle, so it should be the same, every noon. But this phase (ϕ = 1.5) is *not* the same as that of the previous noon (ϕ = 0.5).

Or is it? We could say that ϕ = 1.5 is "one-and-a-half cycles," or we could say that it is "halfway through the *next* cycle." Since we are not interested in which cycle, we ignore the "next" part, and say "halfway"; thus the phase is 0.5.

In fact, whenever we compute a phase, we can make it into a "standard" phase by simply *ignoring* which cycle. If the phase is ϕ = 3.11 (a little more than three cycles), we are 11% of the way through *three cycles later*. We will ignore the "three cycles later" part, note that we are 11% of the way through a cycle, and say the phase is 0.11. All *standard* phases are between 0 and 1.

When phase is expressed in cycles, it is easy to identify which cycle: it is just the *integer* part of the phase, or cycle number. For a phase ϕ = 3.11, the integer part (3) tells us that we are dealing with cycle 3, and 0.11 of 3.11 tells us we are partway through cycle 3. Since a standard phase ignores this, we can simply ignore the integer part of the phase (telling us which cycle). What really counts is the *decimal* part of the phase (how far into the cycle). So we will modify the above equation, and say that the (standardized) *phase* is the decimal part of what we had before:

$$\phi = \text{decimal part of } \left[\frac{t - t_0}{P}\right]$$

What this means is that a phase of 1 (start of next cycle) is really the same as a phase of 0 (start of this cycle) or a phase of 3, or 17, or 256. Any two phases which differ by an integer are really the same phase. Phase 1.5 is the same as phase 0.5, phase 12.336 is the same as phase 0.336, and yes, 0 = 1.

This may seem strange, but it is actually mathematically sound. We are simply taking all numbers *modulo 1*. We can still do arithmetic, still compute numbers, but we always ignore the integer part of whatever we end up with. This kind of arithmetic is known as *modular arithmetic*. In modular arithmetic, modulo 1, we can quite validly state the following equation:

$$0 = 1 = 2 = 3 = 4 = ...$$

Negative Phases

One thing to be careful of is *negative* phases. For the clock example (P = 24 and t_o = 0), let us compute the phase at 6 AM three days *previously*. In this case the time is t = -66 (it is 66 hours before our epoch), so the phase is:

$$\phi = \frac{-66 - 0}{24} = -2.75$$

To convert this to a standard phase, we cannot "just ignore" the –2 and call it 0.75. If we did that, we would also be ignoring the minus sign. So we will just remember that we are doing arithmetic modulo 1, which allows us to add or subtract *any* integer without really changing the result. Let us add 3: $\phi = -2.75 + 3.00 = 0.25$. This does fall in the range 0 to 1, so this is the standard phase.

Folded Light Curves

Take a look at the following light curve of the Cepheid-type variable X Cyg (Figure 12.1). All observations were made by AAVSO observer LX. There are enough data that we can see an up-and-down variation, which turns out to be periodic with a period of 16.285 days. Still, there are only a few observations for each cycle, so it is difficult to tell exactly what the *shape* of a cycle is.

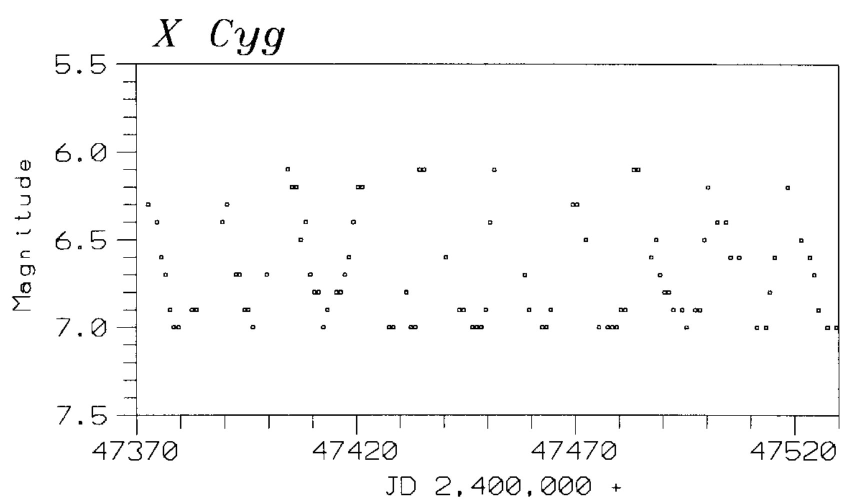


Figure 12.1

It would be nice if we could superimpose all the cycles on top of each other. We would like to plot each data point, but instead of plotting the time, we would like to plot "how far it is into the cycle." That way, all the cycles will be "folded" on top of each other, and we may have enough data to give us an accurate picture of what the cycle looks like. We already have a name for "how far into the cycle": we call it the *phase*. For a variable star, we can do exactly the same thing we did with the clock. Find the period P, choose an epoch t_0, and we can compute the standard phase for any time t. Then we can plot a light curve, but instead of plotting magnitude as a function of *time*, we will plot magnitude as a function of *phase*. This will give us what is called a *folded light curve,* or *phase diagram.*

Let's use the period 16.285 days, and choose as a starting point JD 2,447,400 (an arbitrary choice). Then we can take each observation and convert the time into phase. Plotting brightness as a function of phase, we have the following folded light curve (Figure 12.2):

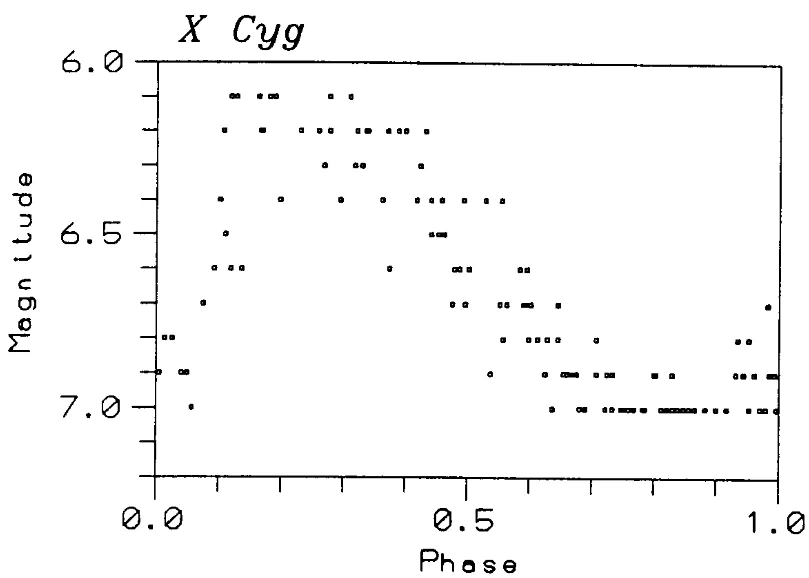


Figure 12.2

Now we *can* see what the shape of the cycle is. There is a very rapid rise from minimum to maximum, followed by a much slower decline from maximum to minimum.

Core Activity 12.2: Folded Light Curve of the Variable Star SV Vul

You are now ready to construct a folded light curve, or phase diagram, using the observations of a Cepheid variable given in the following table.

Table 12.1
SV Vul Magnitude Measurements (1987–1989)

Julian Date	Mag.	Julian Date	Mag.
2447011.6	7.0	2447458.5	6.9
2447023.6	7.5	2447475.5	7.8
2447040.6	7.9	2447492.5	7.9
2447066.5	7.4	2447505.5	7.2
2447091.4	7.0	2447529.5	7.9
2447103.6	7.2	2447707.6	7.9
2447124.6	7.8	2447722.6	6.7
2447171.5	7.9	2447747.6	7.8
2447308.6	7.9	2447769.5	6.8
2447338.6	7.8	2447778.5	7.1
2447374.6	7.0	2447800.5	7.9
2447390.6	7.9	447821.6	7.0
2447404.5	7.8	2447832.5	7.5
2447413.5	6.8	2447848.5	7.9
2447421.5	7.2	2447857.4	6.8
2447444.5	7.9	2447868.5	7.2

1. Construct a graph with the magnitude on the vertical axis and phase on the horizontal axis. Determine the appropriate magnitude scale from the data in Table 12.1. Since all standard phases are between 0 and 1, choose a scale for the phase axis which goes from 0 to 1.

2. We defined the phase as the decimal part of [$(t–t_o)/P$], where t_o is the epoch and P is the period. Take the JD of the very first observation as the epoch, so t_o = 2447011.6. Then the first observation occurs at the start of the cycle (we chose our epoch that way), so we already know the phase of the first observation: it is at phase 0 (start of the cycle). The magnitude of the first observation is 7.0, so plot a point on your graph at phase 0 and magnitude 7.0.

3. For all the other observations, we apply our formula for computing phase. First we take the time of the observation and subtract the epoch time t_0. For the 2nd data point, this gives

   2447023.6 (time of observation t)
   – 2447011.6 (time of epoch t_0)
   12.0 (time difference)

   Then we divide by the period P. For SV Vul, the period is $P = 44.8$ days. This gives

   $12.0 / 44.8 = 0.2679$

   Then we take the decimal part of what we get. Since this result is already between 0 and 1, it is already a standard phase. So for observation #2, the phase turns out to be 0.2679. For plotting purposes, we can round this off to 0.27.

   Repeat this process for every data point, computing the standard phase. When you have computed all the phases, plot each data point at the correct phase and magnitude.

4. Draw a smooth curve showing the trend of the data. Do most of the data lie near this smooth curve? This is a test of the period. Lots of scatter with no obvious trend would show that the measured period is not correct. The correct period should produce a phase diagram whose scatter is about the same as the scatter in the raw data (usually about 0.2 magnitude).

Double Your Fun

Look again at the folded light curve of X Cyg (Figure 12.2). It is a little difficult to see the behavior near minimum, because the picture is "broken" at phase 0 = 1, leaving a gap in the graph. It would be nice if we had a clear picture of the entire cycle, with no breaks.

We can, if we use the fact that phase is a modular quantity, modulo 1. So a phase of 0 is the same as a phase of 1, and the same as a phase of -1. A phase of 0.133 is the same as a phase of 0.133 – 1 = -0.867. A phase of 0.58 is the same as 0.58 - 1 = -0.42. For each time, let us compute not just one phase, but *two* phases. We will compute the standard phase, which is always between 0 and 1, and we will also compute the "previous cycle phase (ϕ')," which will be between -1 and 0. If the standard phase is ϕ, then the "previous cycle phase" is $\phi' = \phi - 1$.

Note that this "previous-cycle phase" will end up being negative. We already learned how to change a negative phase into a standard phase. Now we are changing a standard phase into a negative phase! But not just any negative phase will do. A proper "previous-cycle phase" has to fall in the range of -1 to 0, just as a proper "standard phase" must fall in the range 0 to 1. It is easy to compute, if you just remember to take the standard phase and subtract 1 to give the proper "previous-cycle" phase. **Always compute the standard phase first, then subtract 1 to get the previous-cycle phase.**

We will plot magnitude as a function of phase, but we will plot each data point at *both phases*, the standard phase and the "previous-cycle" phase. In effect, we will be plotting each data point *twice*, and since our phases now run from -1 to +1, we will have a nice picture of not one, but *two* complete cycles. Now it is easy to see what the star is doing at any point of its cycle, because we have an unbroken graph (Figure 12.3):

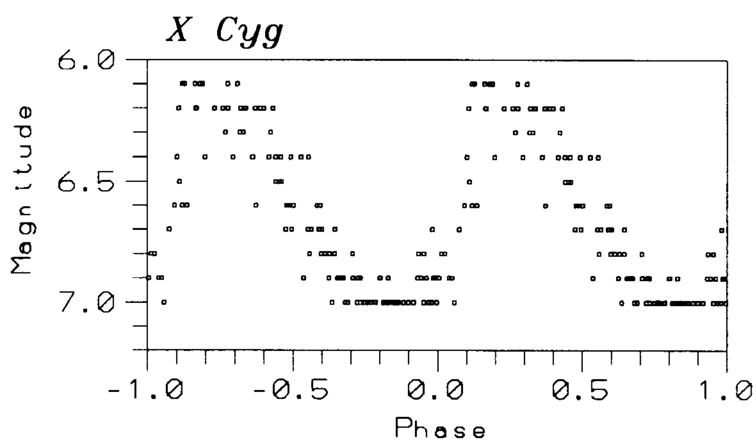


Figure 12.3

When astronomers plot folded light curves, they almost always plot two complete cycles, with phase extending from -1 to +1, in order to give a clear picture of the shape of the entire cycle.

Core Activity 12.3: Another Folded Light Curve of SV Vul

Table 12.1
SV Vul Magnitude Measurements (1987–1989)

Julian Date	Mag.	Julian Date	Mag.
2447011.6	7.0	2447458.5	6.9
2447023.6	7.5	2447475.5	7.8
2447040.6	7.9	2447492.5	7.9
2447066.5	7.4	2447505.5	7.2
2447091.4	7.0	2447529.5	7.9
2447103.6	7.2	2447707.6	7.9
2447124.6	7.8	2447722.6	6.7
2447171.5	7.9	2447747.6	7.8
2447308.6	7.9	2447769.5	6.8
2447338.6	7.8	2447778.5	7.1
2447374.6	7.0	2447800.5	7.9
2447390.6	7.9	447821.6	7.0
2447404.5	7.8	2447832.5	7.5
2447413.5	6.8	2447848.5	7.9
2447421.5	7.2	2447857.4	6.8
2447444.5	7.9	2447868.5	7.2

You are now ready to construct a second phase diagram of SV Vul, this time computing two phases for each point, and plotting a folded light curve showing two complete cycles.

1. Construct a graph with magnitude on the vertical axis and phase on the horizontal axis. Determine the appropriate magnitude scale from the data in Table 12.1. Since we will be plotting two cycles in our folded light curve, our phases will run from -1 to +1. Choose a scale for the phase axis which goes from -1 to 1.

2. We defined the phase as the decimal part of [$(t-t_o)/P$], where t_o is the epoch and P is the period. Take the JD of the very first observation as the epoch, so t_o =2447011.6. Then the first observation occurs at the start of the cycle (we chose our epoch that way), so we already know the phase of the first observation: it is at phase 0 (start of the cycle). The magnitude of the first observation is 7.0, so plot a point on your graph at phase 0 and magnitude 7.0.

3. For all the other observations, we apply our formula for computing phase. First we take the time of the observation and subtract the epoch time t_0. For the 2nd data point, this gives

2447023.6	(time of observation t)
– 2447011.6	(time of epoch t_0)
12.0	(time difference)

   Then we divide by the period P. For SV Vul, the period is P = 44.8 days. This gives

   12.0 / 44.8 = 0.2679

   Finally, we take the decimal part of what we get. Since this result is already between 0 and 1, it is already a standard phase. So for observation #2, the phase turns out to be 0.2679. For plotting purposes, we can round this off to 0.27.

   Repeat this process for every data point, computing the standard phase for each data point.

4. Now compute the "previous cycle phase" for each data point. To do so, simply take the "standard phase" you just computed, and subtract 1. You now have two phases for each data point, a standard phase between 0 and 1, and a previous-cycle phase between -1 and 0.

5. Plot each data point at its correct magnitude, and at *both* phases (so each observation gives two points on the graph).

6. Draw a smooth curve showing the trend of the data. You should be able to discern two complete cycles of variation in the graphs. Do most of the data lie near this smooth curve? This is a test of the period. Lots of scatter with no obvious trend would show that the measured period is not correct. The correct period should produce a phase diagram whose scatter is about the same as the scatter in the raw data (usually about 0.2 magnitude).

Start from the Top

For X Cyg in Figure 12.1, we chose as our epoch, or starting point, JD 2,447,400, just because it was convenient. However, astronomers prefer to choose an epoch so that the *maximum occurs at phase zero*. So let us take as our epoch the time of one of the maxima, JD 2,447,403.0. Then our folded light curve has its maximum right at phase 0 (Figure 12.4):

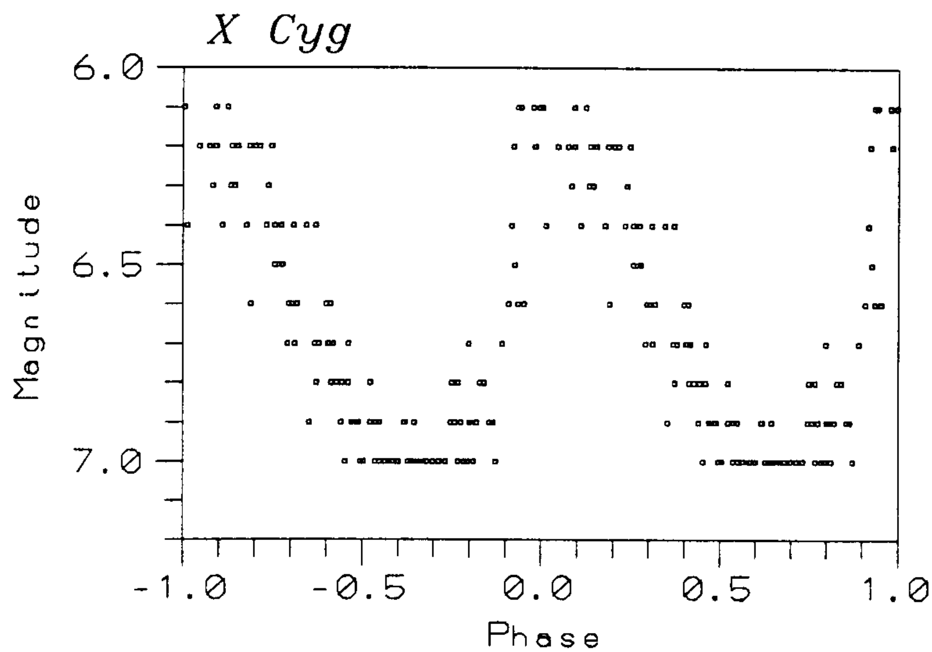


Figure 12.4

This is the *standard folded light curve* for a variable star. It plots two complete cycles, with phase running from –1 to +1, and the epoch is chosen so that maximum occurs at phase zero.

There is one exception to this rule: eclipsing binary stars. For eclipsing binaries, the *minimum* brightness (which usually occurs in the middle of the eclipse) is the part we are really interested in, so we choose the epoch so that phase zero is *minimum* rather than maximum.

Core Activity 12.4: Yet Another Folded Light Curve of SV Vul

Table 12.1
SV Vul Magnitude Measurements (1987–1989)

Julian Date	Mag.	Julian Date	Mag.
2447011.6	7.0	2447458.5	6.9
2447023.6	7.5	2447475.5	7.8
2447040.6	7.9	2447492.5	7.9
2447066.5	7.4	2447505.5	7.2
2447091.4	7.0	2447529.5	7.9
2447103.6	7.2	2447707.6	7.9
2447124.6	7.8	2447722.6	6.7
2447171.5	7.9	2447747.6	7.8
2447308.6	7.9	2447769.5	6.8
2447338.6	7.8	2447778.5	7.1
2447374.6	7.0	2447800.5	7.9
2447390.6	7.9	447821.6	7.0
2447404.5	7.8	2447832.5	7.5
2447413.5	6.8	2447848.5	7.9
2447421.5	7.2	2447857.4	6.8
2447444.5	7.9	2447868.5	7.2

You are now ready to construct a "standard" phase diagram. Estimate the maximum by inspecting the data in Table 12.1, and choose an epoch so that the maximum occurs at phase 0.

1. Construct a graph with magnitude on the vertical axis and phase on the horizontal axis. Determine the appropriate magnitude scale from the data in Table 12.1. Since we will be plotting two cycles in our folded light curve, our phases will run from –1 to +1. Choose a scale for the phase axis which goes from –1 to 1.

2. We defined the phase as the decimal part of $[(t - t_o)/P]$, where t_o is the epoch and P is the period. The brightest of all the observations is the estimated magnitude of 6.7 on JD 2447722.6. Take this as a rough estimate of the time of maximum, and use it as your epoch: t_o = 2447722.6.

3. For each observation, apply the formula for computing phase. First we take the time of the observation and subtract the epoch time *t0*. For the 1st data point, this gives

2447011.6	(time of observation *t*)
– 2447722.6	(time of epoch *t0*)
-711.0	(time difference)

Then we divide by the period *P*. For SV Vul, the period is $P = 44.8$ days. This gives

$$- 711 / 44.8 = -15.8705$$

Finally, we take the decimal part of what we obtain. Since this result is negative, we remember to add an integer to make the sum fall between 0 and 1. Adding 16, we get the standard phase as $\phi = 0.1295$. Repeat this process for every data point, computing the standard phase.

4. Now compute the "previous cycle phase" for each data point. To do so, simply take the "standard phase" you just computed, and subtract 1. You now have two phases for each data point, a standard phase between 0 and 1, and a previous-cycle phase between –1 and 0.

5. Plot each data point at its correct magnitude, and at *both* phases (so each observation gives two points on the graph). This is the standard folded light curve.

6. Draw a smooth curve showing the trend of the data. You should be able to discern two complete cycles of variation in the graphs. Does the maximum lie at phase 0? This is a test of the epoch; if the maximum is noticeably different from phase 0, then the epoch is not quite correct. Do most of the data lie near this smooth curve? This is a test of the period. Lots of scatter with no obvious trend would show that the measured period is not correct. The correct period should produce a phase diagram whose scatter is about the same as the scatter in the raw data (usually about 0.2 magnitude).

The Discovery of SS Cygni

(Adapted from a paper entitled "The Centennial of the Discovery of SS Cygni " by Martha L. Hazen, published in the Journal of the AAVSO*, Volume 26, 1997, pp. 59–61. Dr. Hazen is the Curator of Astronomical Photographs at Harvard College Observatory. Additional information about SS Cygni was provided by the technical staff of the AAVSO.)*

In the *Harvard College Observatory Circular* No. 12, signed by Edward C. Pickering and dated November 2, 1896, there appeared a listing entitled "New Variable Stars in Crux and Cygnus." The last paragraph of the listing reads:

> In addition to the above objects a star in the constellation Cygnus, whose approximate position for 1900 is R.A. = $21^h38^m.8$, Dec. $+43^o8$ has been found to be variable by Miss Louisa D. Wells. Its period appears to be about 40 days and its photographic brightness varies from 7.2 to fainter than 11.2, an unusually large range for a variable having so short a period.

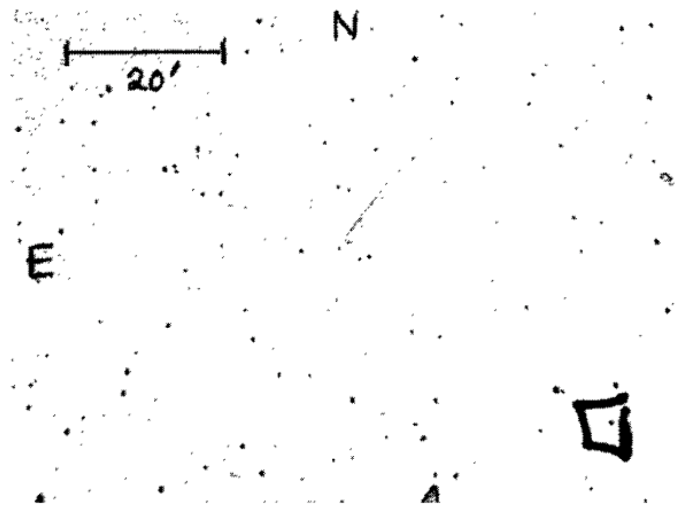


Figure I a. SS Cygni near minimum.

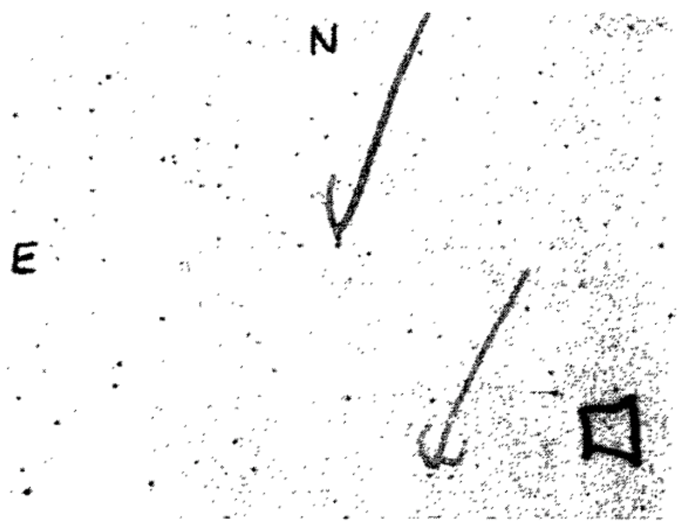


Figure 1 b. SS Cygni in outburst.

Miss Louisa D. Wells was one of the women "computers" hired by E.C. Pickering to work with the Harvard photographic plates. All the original research notebooks kept by the "computers" are stored at the Harvard University Deposit Library, and a complete list of the notebooks is available at the Plate Stacks of the Harvard Observatory. Curiously, Miss Wells's notebooks, necessary for her work, are not in the collection. Her supervisor at the time made no mention of the discovery.

The earliest plate intentionally taken of SS Cyg is plate I 15990, taken with the 8-inch Draper refractor located on the grounds of the Observatory in Cambridge, MA. The plate was taken on September 23, 1896, according to the record book. The entry in the "Object" column says "Susp. var." and the first word is crossed out and "L.D. W.'s" written above.

The earliest plate still extant was taken on September 24, 1890 (see Figure I a; a hand drawn arrow points to SS Cyg in roughly the center of the photo), when SS Cyg was at or near minimum. Figure 1 b is a plate taken on October 30,1890, when the star was in outburst.

The complete details of the discovery of SS Cyg may never be known, but with its discovery began a long period of visual observations so complete that an outburst of this well-studied star has never been missed.

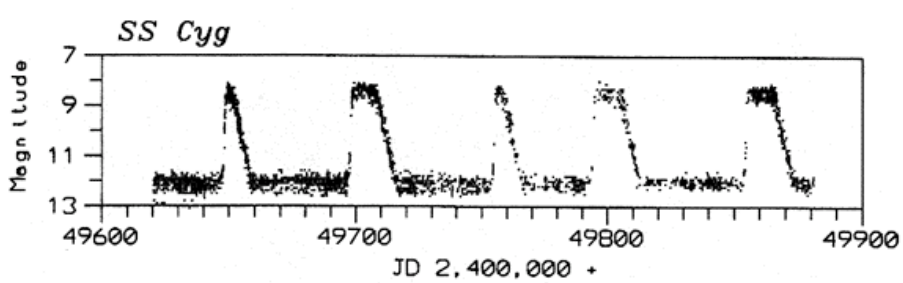
SS Cygni is one of the most famous variables in the sky. It is an eruptive variable of the U Geminorum type. Like most U Gem stars, SS Cyg is part of a spectroscopic binary system-a close system with a cool main sequence star orbiting a white dwarf. The cool star loses matter from its surface, which accumulates in an accretion disk around the white dwarf. For as yet not well understood reasons, the disk becomes unstable and the disk material spirals towards the white dwarf triggering a small nova-like eruption in this dwarf nova system. (Hence the name of dwarf novae given to U Gem variables.) Every few weeks, on average, SS Cyg brightens from magnitude 12 to magnitude 8. At minimum, it can be observed with a small telescope. At maximum, it can be observed with binoculars.

SS Cyg is a spectroscopic binary consisting of dwarf G and subdwarf B spectral class components. The orbital period is 6 hours and 38 minutes; however, no eclipses are observed so the plane of the orbit must be considerably inclined to the line of sight. In other words, from our perspective here on Earth, one star does not pass in front of or behind the other. Spectroscopically, the system alternates between the spectrum of the dwarf G (5520K) during minimum and the subdwarf B component during maximum. As the star rises to maximum, the spectrum changes progressively to that of the subdwarf B star (12,000K).

Visual observations of SS Cyg have become particularly important since it was discovered from satellite observations that SS Cyg is an extreme ultraviolet, and X-ray emitter. The satellite users depend on visual observers to tell them when the star is bright and active, and therefore worthy of further observation by satellite. Observations by amateur astronomers are necessary because the period of SS Cyg is not predictable. The rise to maximum is generally quite rapid, but by no means uniform. Some outbursts are quite short in duration. The light curve of other outbursts may be quite flat for several days at maximum. It then gradually slopes until a more rapid decline sets in. Occasionally there is a slight brightening before the actual maximum, and at other times there is a change from a steep slope to a more gradual one during the decline.

The minimum is the star's quiescent time. At times the light curve is quite flat and at other times quite irregular, with occasional rises in magnitude to at least magnitude 10.0 or brighter, preceding regular maximum.

SS Cygni has had over 700 outbursts since its discovery in 1896 and not one outburst has gone undetected. Statistical studies have shown that there are correlations between light curve characteristics such as the brightness and duration of outbursts and the interval between them. If you wish to study these correlations, you may through the HOA Website-request long term data of SS Cygni from the AAVSO International Database.



Activity 12.5: Folded Light Curves of Star X and Delta Cep

1. If you have completed Core Activity 6.5 in Chapter 6 and calculated the period for Star X, you may now use your own estimated period for Star X as the period *P*, and take the brightest single observation as a rough estimate of the epoch t_o, or time of maximum.

2. Using this period and epoch, and your data for Star X, follow the same procedure as in Core Activity 12.4. You will end up with a standard folded light curve of Star X.

3. Study the resulting diagram. Is your period accurate? Is there a lot of scatter? Is the epoch accurate? Is the maximum at (or near) phase 0?

4. Construct another standard folded light curve of Star X, but this time use the class average period as your period *P*. Are the folded light curves the same? Which period estimate do you think is more accurate?

5. If you have observed delta Cep, use your own observational data and estimate the period for delta Cep to construct a standard folded light curve. Was your period accurate?

Core Activity 12.6: VSTAR

Let's say that you suspect that your data are fluctuating with some particular period. You could test this period by using it to construct a phase diagram. If the scatter in the resulting phase diagram is much less than the scatter in the data, then you have evidence that your data are fluctuating with that particular period.

You could even use this strategy to find an unknown period: simply test a *very* large number of periods by constructing phase diagrams. For a Mira-type variable, you might test periods from as short as 100 days to as long as 1000 days, in 0.1-day steps. So you would construct one phase diagram to test the period 100 days, another phase diagram to test the period 100.1 days, another for 100.2 days, etc., all the way up to 1000 days. That's a lot of phase diagrams!

It takes too long to do this by hand, but this kind of work is ideally suited to a computer. This particular period search method is one of the most common in astronomy. It is called the *analysis of variance*, also known as *AOV*. The VSTAR program will search for periods in your data, using AOV. If you find a likely period, it will construct a phase diagram for you.

Run the VSTAR program, and again select the star V Cas. Load the data from JD 2447000 to 2449000. Now hit the <F4> key (AOV (period)). It will ask you for the "number of bins" and suggest the answer 20; this is a good choice in this case, so just hit <ENTER>. You will see the "period analysis" menu.

Select option <F2> (frequency range). When it asks you for the "low frequency to test," enter 0.001, when it asks for the "high frequency to test," enter 0.01, and when it asks for the "frequency resolution," enter a 0 (which tells VSTAR to pick the resolution itself). Finally, when it asks "AOK?," answer "Y" and watch what happens.

VSTAR will test a lot of periods (frequencies), and for each period it tests it computes a *power* level. The power level is a measure of the likelihood that the data are periodic for a particular period. Power levels higher than 10 mean that the data *might* be periodic (or might *not*). As it computes, VSTAR draws a graph for you, showing power as a function of frequency. This is the most basic graph in period analysis: it is called a *periodogram*.

Possible periods show up as large power values, and on the periodogram plot they look like spikes. So when you see a spike in the periodogram, it represents a *possible* period. VSTAR saves the period of the ten tallest spikes in its "top-ten" list so you can access them later. After the periodogram plot is finished, hit the <F4> key. Now VSTAR shows you the top-ten list, giving the period and power level. Can you tell by looking which list entry goes with which spike on the graph?

When you hit <F4>, VSTAR also asks which entries you want to delete. Enter "4-10," so VSTAR will delete entries 4 through 10, leaving only 1, 2, and 3. These are certainly the most likely possible periods. Then give no answer to the "delete" question, and VSTAR will return you to the period analysis menu. Now hit the <F5> key (model the data). VSTAR asks which frequency to include. Enter "1" (to use the #1 period), and VSTAR will construct and display a phase diagram, using that period. VSTAR will ask if you want to "save to a file" (say "N" for no).

CAUTIONARY NOTE: When VSTAR shows you a phase diagram, it does not show two complete cycles, only one. Also, it does not put the maximum at phase zero; it chooses an epoch at random. So be prepared: if you see two complete cycles, it is not because it is a standard folded light curve! It is because your period is too long—you have gotten two cycles per period. Hit <F5> again, and this time choose period #2. You will see two complete cycles, because this is the wrong period! Period #2 is the twice the true period. Model the data again, using period #3, and you will see three complete cycles, because this is also the wrong period; it is three times too long.

When you fold the data using the correct period, you get a nice folded light curve of one cycle. When you fold the data using twice (or three times, four times, or any multiple of) the correct period, you also get a nice folded light curve, but of more than one cycle. So when VSTAR shows many possible periods, it is up to you to look at them, and decide which one is real.

If the true period is P, then its multiples are 2P, 3P, 4P, etc. Even though they give a high power level, they are not the true period, they are *aliases*. Since the frequency is f=1/P, the alias frequencies are 1/(2P), 1/(3P), 1/(4P), etc. So when the data are periodic with frequency f, you will get a spike at frequency f, and at frequencies f/2, f/3, f/4, etc. These alias frequencies are called *subharmonics*; it is a property of AOV that it gives high power levels not only for the true period (frequency), but also for its subharmonics. It is up to the analyst (you!) to decide which is real and which is an alias.

When you have decided which period is real, use that period to model the data. Now when you are asked whether you want to save to a file, say "Y" for yes. Then pick a file name for the saved information. VSTAR will create a file containing both phase and magnitude (just what is needed to plot the standard folded light curve). It will give you two complete cycles, and it will also estimate the maximum, choosing an epoch so that maximum is at phase zero: the standard folded light curve. See the VSTAR manual for the layout of this file. Plot the standard folded light curve for V Cas from JD 2,447,000 to 2,449,000. You will need to use a graphing program for this (there are too many data points to do it by hand!).

Now repeat the entire procedure, but instead of using the data from JD 2,447,000 to2,449,000, use the data from JD 2,440,000 to 2,442,000. Now you have *two* folded light curves, covering two different time periods. They show the *average* shape of the light curve during those time intervals. Are they the same? Is the average maximum brightness the same? Minimum? Is the *shape* of the average light curve the same?

Finally, a most interesting question: why does VSTAR show a spike at periods which are multiples of the true period (in other words, why does AOV respond to subharmonics)?

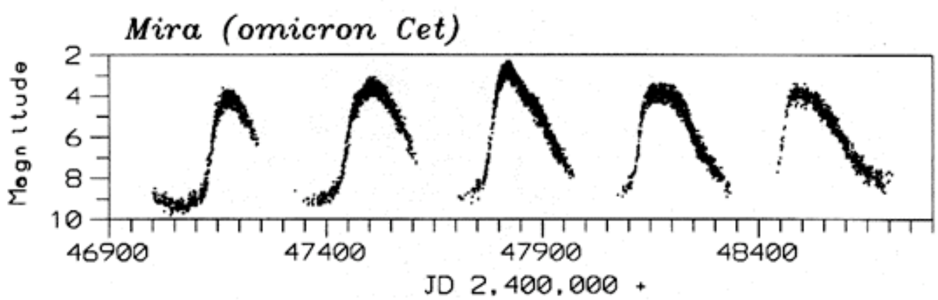
SPACE TALK

Mira stars are long-period, pulsating red giants of approximately the Sun's mass that have entered the final evolutionary stages of their existence and will eventually become white dwarfs. Miras have nearly exhausted the supply of hydrogen in their cores. Their cores are very dense and are composed mostly of oxygen and carbon (products of helium fusion). Just outside the core, a shell of hydrogen is still being converted to helium, and this layer of helium builds up on the surface of the core. Every few thousand years enough helium builds up and then ignites, creating even more carbon and oxygen. When the helium-burning begins, the shell rapidly expands and the hydrogen-burning turns off. This is known as **helium flash**. When most of the helium is consumed, the flash ends, the shell shrinks, and hydrogen-burning resumes. This process can happen over and over again for 50,000 to 100,000 years before the outer layers are thrown off to form a planetary nebula. The core remains as a white dwarf, its nuclear fires finally out.

1996 marked the 400th anniversary of the discovery of the first Mira-type variable star. It is located in the constellation Cetus the Whale, and is known as Omicron (o) Ceti, or Mira, and was the very first known periodic variable star. David Fabricius, a clergyman and amateur astronomer in Friesland, Germany, noticed the "new" star in Cetus on August 13, 1596. Fabricius checked every star catalogue, atlas, and globe that existed and saw that the star was not listed. He observed Mira again at the beginning of September and watched it fade below naked-eye visibility during the middle of October. Fabricius assumed that his star was a nova similar to Tycho's nova discovered in Cassiopeia (Supernova of 1572). As a result, Fabricius did not check the star after it dimmed, because novae do not brighten more than once. He did not notice the star again until 1609. Fabricius might have studied his star more systematically with the recent development of the telescope, but unfortunately he met an untimely end when he was murdered by a member of his own parish in 1617. (He had recently announced from the pulpit that he knew which member of his parish had stolen one of his geese!)

Mira was listed in Johann Bayer's 1603 star atlas *Uranometria* as an ordinary 4th-magnitude star labeled Omicron. Another German astronomer, John Holwarda, also from Friesland, discovered in the winter of 1638–39 that Mira had brightened, and realized that it would probably repeat the increase in magnitude. Every maximum since 1638 has been observed except for those times when Mira's apparent position in the sky appeared too close to the Sun to be seen, which occurs from April to the end of June each year. In 1667, Ismael Boulliau announced that Mira's variations were periodic and that the star brightened every 333 days. The period is 331.96 days, as published in the 4th edition (1985) of the ***General Catalogue of Variable Stars*** (GCVS). Johannes Hevelius, who named the star Mira ("the wonderful"), began observing it regularly in 1648, and in 1662 published a pamphlet about the star entitled "Historiola Mirae Stellae" ("Brief History of the Wonderful Star"). In 1926, Sir Arthur Eddington gave the correct explanation for the behavior of Mira-like variables—that these stars pulsate the same way Cepheid variables do, but with longer periods because of their more distended physical size and lower surface gravity.

Mira typically ranges in brightness from magnitude 9.3 to 3.4. In some cycles Mira brightens to a brilliant 2nd magnitude, and in other cycles barely reaches 5th magnitude. The brightest maximum on record occurred in November of 1779, when William Herschel observed Mira to be almost as bright as Aldebaran (magnitude 0.9). Maxima observed since 1906 by members of the AAVSO have ranged from 2.4 to 4.9, and minima have ranged from 8.4 to 9.7. The period also displays irregularities, with maxima arriving three weeks earlier or later than predicted. Unlike most Mira-type variables, omicron Ceti is a double star system. It has a 10th-magnitude companion, a hot dwarf called VZ Ceti. This companion was seen for the first time in 1923, although it had been detected by spectroscopic methods 5 years earlier. The irregularities in Mira's variations are obvious in the AAVSO light curve shown below, based on more than 17,000 observations by amateur astronomers over the past 25 years.



A star pulsates because it is not in **hydrostatic equilibrium**: the force of gravity acting on the outer mass of the star is not quite balanced by the interior radiation pressure pushing outwards. If a star expands as a result of increased gas pressure, the material density and pressure decrease until the point that hydrostatic equilibrium is reached and then overshot, owing to the **momentum** of the expansion. Then gravity dominates, and the star begins to contract. The momentum of the infalling material carries the contraction beyond the equilibrium point. The pressure is again too high, and the cycle starts over again. The system acts as an **oscillator**. However, with loose atmospheric layers of gases, the oscillations get out of sync, or phase, with one another and set the stage for chaotic motions. Energy is dissipated during such pulsation (analogous to losses caused by friction forces), and eventually this loss of energy should result in a damping or lessening of the pulsations. The prevalence and regularity of pulsating stars imply that the dissipated energy is replenished in some way. This kind of statistical conclusion requires very long runs of data, such as those collected by the AAVSO.

Mira stars are not entirely predictable, and individual cycle lengths can be several weeks shorter or longer than the mean. Miras also undergo small, longer-lasting changes in their periods which are revealed by sophisticated statistical techniques. Subtle departures from regular behavior are characteristic of these stars. However, a handful of Miras go way beyond subtle changes, instead exhibiting extreme period changes that indicate radical physical changes within the interior of the star. One such star is R Hydrae, the third Mira-type variable discovered, located ~325 light-years away. (The second Mira-type variable discovered was chi Cygni in 1686.) R Hydrae missed discovery twice, once by Johannes Hevelius who recorded it in his star catalog simply as a 6th-magnitude star, and once by Geminiano Montanari, the Italian astronomer who worked at Bologna and discovered the variability of Algol in 1669. Montanari noticed R Hydrae at naked-eye brightness in April 1670, noted that it was not listed in Bayer's *Uranometria* star catalog, and added it to his copy by hand. Montanari's copy of *Uranometria* ended up in the possession of Giacomo Maraldi, who worked with his uncle, Giovanni Cassini, at the Paris Observatory. (Cassini discovered the division in the rings of Saturn which now bears his name.) Maraldi saw the handwritten notation in the star catalogue and began searching for the star in 1702, finally discovering it two years later at 4th magnitude.

The most remarkable aspect of R Hydrae is its slow but dramatic shortening of period, from 495 days during early observations to only 389 days during the last 60 years. For the first century of observation, R Hydrae's period decreased at a nearly constant rate, from 485 days/cycle around the year 1800 to 400 days/cycle around 1910. From 1923 to 1935, the period slowed to 415 days, then suddenly increased again. Since 1937, the period has remained constant at 389 days. The century of steady decrease in R Hydrae's period is consistent with theoretical calculations of what happens to a pulsating red giant after helium flash. R Hydrae is still recovering from such an event.

Amateur variable star observers usually do not get to actually observe the dramatic changes that evolving red giants undergo. One exception is that of T Ursae Minoris, a red variable in the bowl of the Little Dipper. After decades of constant periodicity, the pulsations of T Ursae Minoris started to increase drastically, probably due to the earliest stages of helium flash. Before 1980, the period ranged from 310 to 315 days, but since 1980, it has decreased steadily to 274 days. If the theoretical models of stellar evolution are correct, T Ursae Minoris should continue shortening its period until it reaches a minimum period of 200 days (around the year 2030), after which its period will once again lengthen.

POSTER TALK: "Theoretical Glue": Understanding the Observed Properties of Miras with the Help of Theoretical Models

by Dr. Lee Anne Willson

Lee Anne Willson, Professor of Astronomy at Iowa State University, is an internationally-recognized expert on Mira variable stars. The following is an adaptation of a paper she presented at a special scientific conference on Mira stars sponsored by the AAVSO in 1996. The full text, including additional references and bibliography, can be found in *The Journal of the AAVSO*, Volume 25, No. 2, pages 99–114.

1. Introduction

Each observational study of Mira variables has as its goal to determine some quantities describing these stars. However, observations alone do not give us an understanding of what we are seeing. Theoretical models are needed both to connect what is observed to the qualities of the star, and to link the various measurements into one coherent picture of its nature. It is in this sense that a good model is a kind of "glue" holding the picture together.

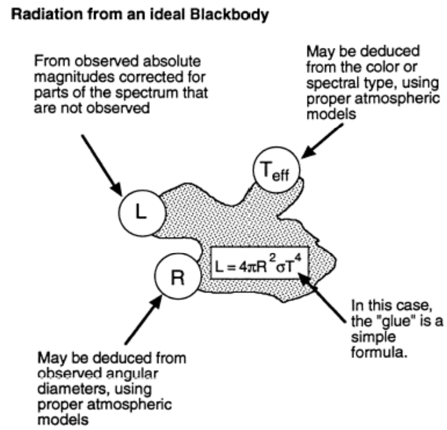


Figure 1. The definition of effective temperature (T_{eff}) illustrating the concept of theory as "glue."

Figure 1 illustrates the concept of "theoretical glue" by showing how the luminosity L, the radius R, and the effective temperature T_{eff} are related by the theoretical (and lab-tested) model of a blackbody "perfect radiator." Such a perfectly radiating surface emits power per unit surface area that increases as the fourth power of T (in Kelvins), so a doubling of the temperature gives a 16-fold increase in the total amount of radiation (light, infrared, ultraviolet, X-rays and so on) coming from each patch of the surface. If stars were, in fact, ideal blackbodies, then their radiation would be completely known and the problem of relating L, R, and the temperature of the surface would be trivial. However, real stars are gas spheres; we can only see into the atmosphere on the average down to the apparent surface, the photosphere. We define the "effective" temperature T_{eff} as the temperature of a blackbody of the same size as the star that radiates the same total power, L. This gives the equation $L = 4\pi r^2 \sigma T_{eff}^4$ with $\sigma = 5.670 \times 10^{-8}$ watts per square meter per second per $(\text{Kelvin})^4$. Moreover, the stellar atmosphere is not all at one single temperature, and we see to different depths at different wavelengths; as a result, the spectrum we see has high and low spots compared with an ideal blackbody spectrum. Typically, the effective temperature is close to the gas temperature at the photosphere, but is not identical to it. We have to use a theoretical model atmosphere to relate effective temperature to the spectrum we see and to the temperature at the photosphere.

For Miras and other pulsating variables, even more than for most stars, the process of translating "what is actually observed" into "what the star is really like" can lead the incautious investigator astray. For example: To get the luminosity—the total power output, energy per second—we observe the visible part of the spectrum and, if we are lucky, also the near-infrared and perhaps the ultraviolet, in detail or using broad-band photometry. We then use a model of some sort to estimate how much light we are missing in the parts of the spectrum that we can't see, and finally we correct for distance. How good our final result is will depend on how good the model is that we use to fill in the "missing bits," as well as on how much of the spectrum we could actually observe and how accurately we know the distance. The expected (and sometimes observed) spectra of Miras are very far from the simplest case—a blackbody spectrum—so detailed models are essential. Worse, the molecules that produce some of the deepest spectral features in Miras are also found in Earth's atmosphere, and so we are selectively less likely to observe the depressed parts of the spectrum.

2. Classical model atmospheres

The calculation of a classical stellar atmosphere begins with a choice of stellar parameters—for example, composition, effective temperature, and surface gravity. The propagation of energy from the interior of the star through the atmosphere and into space is then calculated, taking into account the effects of the different atoms, ions, and perhaps molecules that can absorb and emit light. The result of a classical atmosphere calculation may include any or all of a predicted spectrum, a model for the pattern of brightness that you would see if you could get close to the star, and predicted values for the photometric colors. Such models play a key role in the determination of the luminosities of stars, and also in the derivation of their radii.

To get an estimate for the radius or diameter of a star, we may use an interferometer or a lunar occultation to get a pattern of fringes that may be interpreted by using a model for the brightness pattern on the star. Or, we may try to relate the appearance of the spectrum to the effective temperature T_{eff} using a detailed model atmosphere, and then deduce R from L and T_{eff}. If these methods give the same answer, it increases our faith that the model is close to describing what happens on the star.

To find the composition of the atmosphere, the line spectrum is analyzed using a stellar model atmosphere. Thirty years ago, most such calculations were made using some reference model atmospheres and looking for differences using methods such as the "curve of growth." Today, it is possible to carry out most analyses by making model atmospheres with a range of compositions and selecting the composition pattern that produces a spectrum that best matches the observations.

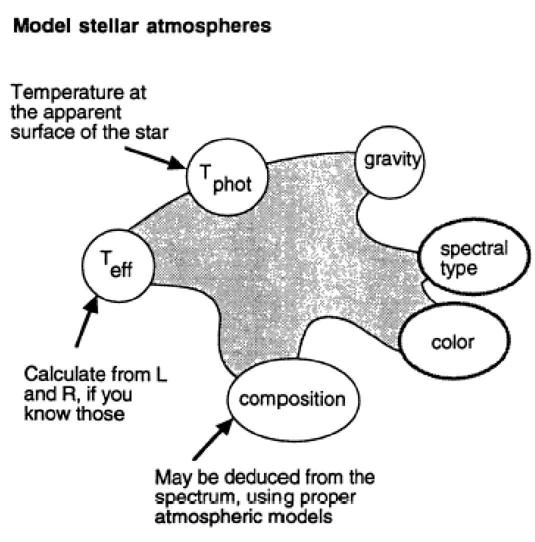


Figure 2. Classical model atmospheres as "glue" for the colors and spectra.

Figure 2 illustrates how a classical stellar model atmosphere glues together observable and non-observable quantities for stars. In a classical model atmosphere there is no net outflow of matter—no stellar wind—and there are no systematic motions, such as one might get from pulsation. Obviously, this is not going to work perfectly for modeling Miras! Also, in classical atmospheres, each part of the atmosphere is in radiative equilibrium—meaning that the radiant energy flowing into a sample volume of the gas per second exactly equals the radiant energy flowing out of the same sample volume per second. In more modern model atmospheres, other forms of energy are also considered, and energy is allowed to shift from one form to another—for example, from May be deduced from the spectrum, using proper sound waves to radiation. There are still relatively few models, however, that include dynamical effects and outflows.

One of the important ingredients in a stellar atmosphere model—whether classical or modern—is the surface gravity $g = GM/R^2$, where G is the gravitational constant, and M is the mass of a pulsating star. This combination of M and R turns out to be important for the spectrum because higher gravity compresses the atmosphere more. One can deduce a gravity by computing synthetic spectra for models with a range of surface gravities, and then picking the model whose spectrum best matches the star, as long as the model does produce a good match. This works reasonably well for most non-variable stars, but does not do a good job with the variable ones. Other methods are needed for these, at least for now.

3. Glue from pulsation and evolution studies

In principle it should be easier to derive a value for the surface gravity for a pulsating star, because the material in the atmosphere is moving in response to gravity during much of the cycle. Thus, one might observe the change in velocity over some interval of time and estimate $\Delta v/\Delta t = g$. Because the excursion in radius is large (so g is not the same at all parts of the path) and because pressure forces are also important, the above method typically underestimates g by a factor of five or so. Instead, a dynamical atmosphere model needs to be used to interpret the result. Also, you need a good radiative transfer model to be able to interpret the observed Doppler shift in terms of the motions of parts of the atmosphere, because only part of what you see is moving towards or away from you. To get a meaningful Δv from the Doppler shift requires a fairly detailed model for the atmosphere, and this correction is still rather rough for most variable stars.

There is another way to get a combination of M and R for pulsating stars. A given star is usually able to pulsate only in one or a small number of modes, each with a distinct period associated with it. Detailed models for the interior of a pulsating star can be analyzed to reveal the period(s) that are possible, and these can be related through formulae such as (for example), $P = aR^bM^c$, as is illustrated in Figure 3. (Usually b is between 1.5 and 2, and c is between –0.5 and –1.)

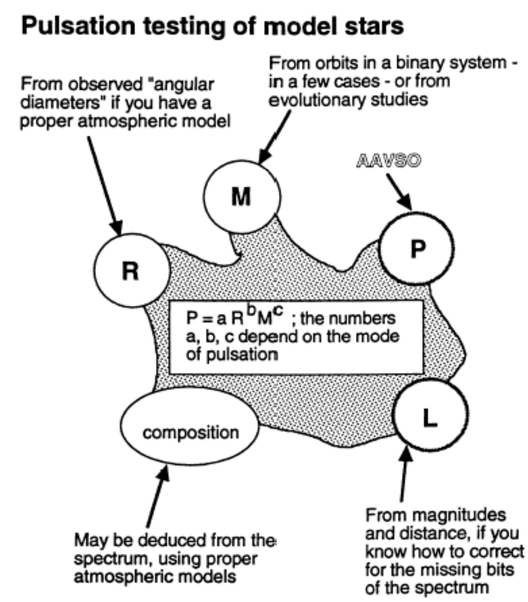


Figure 3. Pulsation periods from model stellar interiors connect evolutionary models to stellar parameters.

There are assumptions that go into this kind of modeling that need to be tested more thoroughly than has been possible so far: for example, the period of pulsation of a star pulsating at full amplitude may not be the same as the period derived looking at very small pulsations in a model for a static star.

A PMR relation is often used to estimate the mass of a pulsating star, M, given its radius, R (which may have been derived from L and T_{eff} or from angular diameter measurements), assuming that one knows the mode of pulsation. It can otherwise be used to determine the pulsation mode(s), if one is confident of M and R from other measures. For most classes of stars this is relatively easy to do, and the results are consistent with whatever else one knows about the stars. However, for Miras the radii and masses are still sufficiently uncertain that this method does not even yield an incontrovertible result about the pulsation mode, much less useful estimates for their masses.

4. Models for stellar evolution

The most important "theoretical glue" in stellar astronomy is the study of how stars evolve. Starting with some composition (assumed to apply throughout the star) and a mass, M, a model is found that obeys relevant physical equations and is in hydrostatic equilibrium. For all but the lowest-mass stars the model will include energy generation by nuclear reactions in the core. These reactions modify the composition at the center, so some time later the star's structure will be a little different and its L and R may also be a little different. By building a sequence of static models that are related by the condition that the change of composition comes from the nuclear reactions, one can model the evolution of the star.

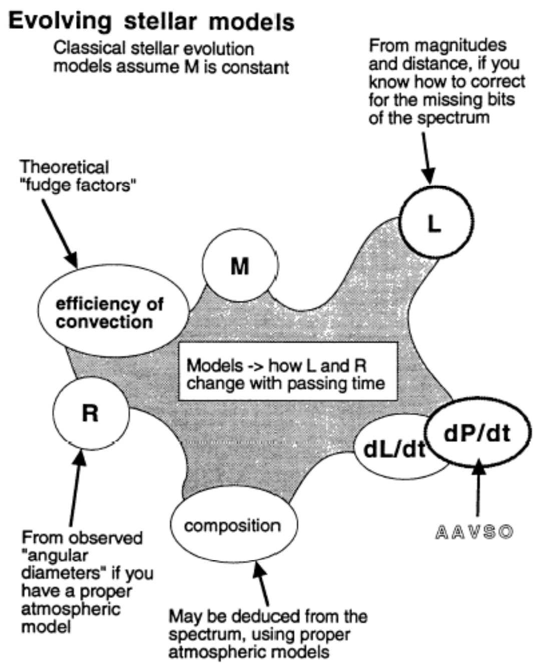


Figure 4. Evolutionary models link stars in different evolutionary states as well as relating stellar properties and (occasionally) rates of change of those properties.

In most evolutionary calculations the mass M is not allowed to change with time, although there are times in the life of a star when the mass decreases as the result of mass loss from the surface. The change in mass that comes from the conversion of mass to energy in the nuclear reactions is almost always small enough to ignore. One time when the mass loss is particularly important is the Mira stage, and this fact is a major reason why Mira models are not yet in a settled state.

Since much of stellar evolution proceeds at constant mass, and since L and T_{eff} are the easiest quantities to estimate directly from observations, we traditionally plot tracks for constant mass stars in a diagram of L versus T_{eff}, one variant of the Hertzsprung-Russell diagram. One may then think of evolutionary tracks as linking L, M, T_{eff} or R, initial composition, and age for the star (Figure 4).

In addition to the problem of how to include mass loss in a realistic way, evolutionary models also suffer from our lack of detailed understanding of convection in stars; of rotation inside stars; of the effects of magnetic fields in stars; and so on. Most astronomers assume that these effects will turn out to be small, but others would not be surprised to learn that some of them affect the "big picture."

5. Dynamical models for the atmospheres of pulsating stars

If we know how a star is pulsating, then we can model the response of the outer parts of the star (the atmosphere) to this pulsation. Figure 5 shows the connections that can be made this way. In practice, L, M, and T_{eff} or R are assumed; also a pulsation period P is assigned (using a PMR relation) and the bottom of the atmosphere is made to move in and out with period P.

Dynamical models require some understanding of the interaction between the gas and the radiation. The pulsation generates waves that compress the gas, heating it. It then cools by radiating away energy, and also by expanding. Depending on the density of the gas, the conversion of internal energy into radiation may be fast (compared with the pulsation time) or slow. Where it is fast, the material cools to roughly the equilibrium temperature that it would have in a static model, and then as it expands it may be refrigerated below the temperature it would have in the static case.* Where the density is lower, the cooling is less efficient; there, the temperature may never fall as low as the equilibrium temperature. Some dynamical model results are very sensitive to the treatment of these processes; mass loss is one example. Since the details of how the gas emits or absorbs radiation at low density involve many non-equilibrium chemical processes, this is definitely one of the frontier areas in dynamical atmosphere modeling.

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*Since the density is highest just after the gas is compressed, there is a region in the atmosphere where it can lose energy to radiation immediately after compression but has a harder time regaining energy near the end of its expansion. We can describe this approximately by saying that the shock is nearly isothermal-it returns to the radiative equilibrium temperature quickly-but the expansion between shocks becomes nearly adiabatic-without gain or loss of energy.

A detailed treatment of the interaction between gas and radiation—the radiative transfer problem—is also required in order to synthesize the spectrum and colors that would be observed, as well as the light curve. So far, there is no model for Miras or other pulsating stars that includes enough detail to do this effectively. However, dynamical models that are now available provide important insight into the motions of the atmospheres and the mass loss rates that result. For example, Bowen's models (Bowen 1988, 1990) have atmospheric motions and conditions that match what we deduce from observations—shocks with velocity amplitudes of 20 to 30 km/s, warm regions in some, dust formation in others, and so on. In fact, the success of dynamical models in matching velocity variations observed in the infrared CO lines is the best evidence we have about the mode of pulsation of these stars—fundamental mode models match well, but overtone models (with larger radius at a given P) are quite far from matching, as was first noted nearly 20 years ago (Hill and Willson 1979).

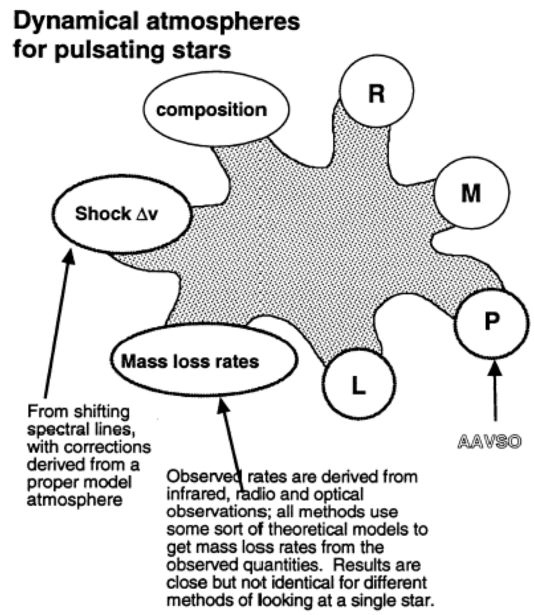

Figure 5. Dynamical model atmospheres are required for pulsating stars, such as Mira variables.

6. Some results of recent "glue" production

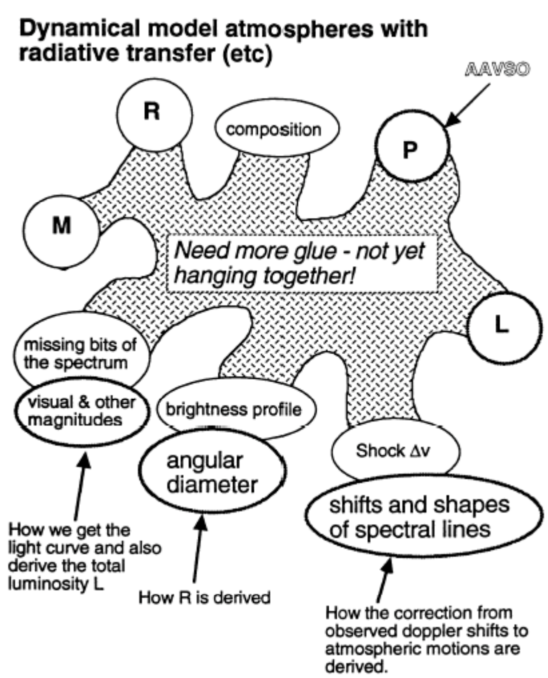

Figure 6. Ideally, radiative transfer, non-equilibrium chemistry, and detailed hydrodynamics are included in the models. In practice, no one model yet includes all the details that are needed to reproduce all the observations.

Bowen's latest grid of dynamical atmosphere models is a collection of models that are constrained by stellar evolution calculations: once L, M, and initial composition are chosen the evolutionary calculations are used to derive R, and then a theoretical PMR relation gives P. This single step of requiring the stars to fall on a single set of evolutionary tracks turns out to make quite a big difference in the way that the mass loss is understood to develop. The choice of which tracks to use is not so important as is the fact that using tracks forces certain relationships between models: For a given mass, as a star increases in luminosity it also increases in radius with (slightly) decreasing T_{eff}. Two stars with the same L and different masses will be separated in T_{eff} or R. Two stars of the same L and M but different composition will also be separated in T_{eff} or R: lower metallicity stars are hotter and smaller at a given L.

Bowen's models, constrained in this way, predict mass loss rates that are very sensitive to stellar L, R, and M. Since all of these parameters vary in a predictable way along a given evolutionary track, we can display the results as mass loss rate M versus L for a given mass, where for a given metallicity L and M together also determine R, T_{eff}, and P as well. Figure 6 shows the result of these calculations for stars whose composition matches that of the Sun.

7. Conclusion: where we need some new glue

Many pieces of the puzzle of Miras and other pulsating variable stars are well-glued together by these theoretical calculations, but some very basic properties remain "unglued." The outstanding problem in the case of the Miras remains the determination of their absolute sizes; here, uncertainties of factors of two or more are still a problem. Clearly we need more and better "glue"—new models that incorporate more of the physics that we already know is important in producing what is observed.

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